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Experimental Investigation on the Chaos-to-Interwell Motion Transfer in a Bistable Beam-Slider Vibration Energy Harvester

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Bistable structures are widely used for vibration energy harvesting due to their wide bandwidths and extraordinary performance. However, the dynamics of bistable structures are complicated, and inter-well, intra-well, chaotic, superharmonic, and subharmonic vibrations may coexist in some frequency ranges. Inter-well vibration is typically the most desired because of its large oscillation amplitude, which means more kinetic energy can be converted into electricity via different energy transduction mechanisms. In this study, a modified bistable beam-slider vibration energy harvester consisting of a cantilever beam and a movable slider on the beam is investigated experimentally. The slider can move along the beam under the combined effect of the inertial and magnetic forces. Moreover, magnetic nonlinearity is incorporated into the beam to achieve bistability instead of the linear or monostable configurations typically found in existing literature studies. The slider trajectory and the bistable cantilever beam time responses show that the slider can help the bistable beam system transfer from the chaotic to the inter-well vibration orbit. The results show that inter-well vibration can be maintained even with disturbance introduced with 3.92 m/s^2 base excitation over the 15 Hz–18 Hz frequency range. The whole transfer process is self-regulating and does not require any external intervention. Therefore, the harvester we designed is self-adaptive, with a substantially broadened operating bandwidth. [DOI: 10.1115/1.4067058]

Keywords: nonlinear structure, bistable, chaos, inter-well vibration, broadband energy harvesting., dynamics, nonlinear vibration, smart materials and structures

1 Introduction

To support the microelectronics and wireless sensors implemented in remote areas and deployed inside machines, harvesting energy from the ambient environment is a more favorable choice than chemical batteries. The unlimited lifespans of energy harvesters effectively reduce the concern and the cost of follow-up maintenance, including recharging and replacing batteries [1]. Vibrations can be converted into electricity through different transduction mechanisms, including piezoelectric, electromagnetic, electrostatic, and triboelectric. Since linear vibration energy harvesters only show high efficiency within narrow frequency bandwidths around resonances, researchers are pivoting toward nonlinear harvesters [2,3]. Bistable vibration energy harvesters are mostly investigated because of their simple structures, wide bandwidths, and large vibration amplitudes. However, the desired inter-well vibration of a bistable harvester often coexists with other responses, such as inner-well vibration and chaos, given the same excitation level and structure parameters [4]. Even though the inter-well vibration orbit is somehow obtained, it is often unreliable, and a small disturbance may lead to orbit jumping.

To maintain the high efficiency of bistable energy harvesters, there are numerous methods to help secure high-energy orbits. Eturk and Inman [5] proposed two direct ways, hand impulse and increasing the excitation force, to assist the bistable energy harvester in entering the high-energy orbit. On the basis of a similar idea, Zhou et al. [6] proposed to generate a disturbance pulse by an additional mass-spring oscillator. The experimental results showed that the bandwidths of the bistable and tristable nonlinear harvesters were broadened to 15 Hz and 12 Hz, respectively, from the original ones of 3 Hz and 5 Hz. Chen et al. [7] connected a linear beam to a bistable beam via a linear spring on their free ends. The potential barrier height of the bistable beam was lowered by the interaction generated by the linear one during vibration. The simulation and experimental results showed that the harvester could exhibit high-energy inter-well vibration in a wide frequency range, even under low-level excitations. Li et al. [8] investigated the characteristics of a bistable energy harvester under the hybrid base vibration and wind excitation. The theoretical analysis and experimental validation showed that the quasi-

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periodic, periodic, and chaotic responses might coexist. Changing the wind speed can help a bistable energy harvester escape from the intra-well vibration. Lan et al. [9] utilized the wind to form the negative damping effect in a bistable energy harvester. The required wind speed for the orbit jump was found to be insensitive to different excitation conditions. This method does not consume the harvested energy at all and is easy to implement. Zhang et al. [10] proposed a bistable electromagnetic vibration energy harvester with an elastic boundary. The numerical and experimental results proved that the potential barrier height was reduced due to the introduction of the elastic boundary, making the favorable inter-well vibration easier to realize. Moreover, they showed that, under the same excitation, not only the amplitude but also the frequency bandwidth of inter-well vibration was enhanced. Wang et al. [11] designed a spring-based bistable energy harvester with two configurations. The two ends of the compressed springs were fixed on the vertical and horizontal beams, respectively, to produce nonlinear restoring forces. The SBEH-sub configuration had four additional magnets to alter the potential well for maintaining inter-well motion. Both the experimental and simulation results showed that the efficiency and operation bandwidth of the SBEH-sub configuration were increased significantly than the SBEH-sup configuration without magnets.

Piezoelectric and electromagnetic transducers in energy harvesters are the elements responsible for converting kinetic energy into electricity. On the other hand, if electrical power is supplied to the elements, the transducers can act as actuators to generate the perturbation to the system. By this principle, Masuda et al. [12] built an interface circuit that could operate in two modes. The normal mode harvests and rectifies energy, while the excitation mode supplies energy to the electromagnets to produce an excitation. As a result, the low-energy orbit of the system is destabilized. Mallick et al. [13] introduced an electromagnetic force to the nonlinear system to switch from the low to high-energy orbit. The basin of attractions explained why the successful rate was sensitive to the control parameters, including the voltage, frequency, and phase angle. Tang et al. [14] developed an active approach for maintaining the high-energy orbit of a bistable energy harvester. The four piezoelectric transducer patches bonded on the cantilever beam were shunted to three subcircuits, including the harvesting, monitoring, and switching-trigger circuits. When the system operates in a low-energy orbit, the damping of the system can be adjusted by the control circuit to capture the inter-well vibration. The experiments validated the jumping from low- to high-energy orbit by applying the method to the prototyped physical model. The output power of the bistable energy harvester was enhanced around 7 Hz. Huang et al. [15] intended to capture the desired high-energy orbit of the buckling beam in a nonlinear energy harvester. By changing the voltage applied on the piezoelectric transducers at the two fixed ends, the buckling level could be changed to modulate the dynamics of monostable and bistable energy harvesters. The theoretical and experimental results revealed that the method could help nonlinear harvesters transfer from low to high-energy orbit with just a little adjustment energy and maintain high efficiency in a wide frequency range. Their more recent work [16] conducted a further study on the jumping details from subharmonic vibration to inter-well vibration, and they reduced the energy consumption during the adjustment process by adding an inductance switch to the circuits. Lan et al. [17] utilized a negative resistance circuit to generate voltage impulses on the piezoelectric element, ultimately for applying mechanical perturbations to monostable and bistable energy harvesters. They proved that it was feasible to help obtain high-energy orbits since the energy requirement was low.

Most of the methods mentioned earlier are active or semi-active. Devices such as actuators, control circuits, and condition monitors are required for completing the transfer process. It is irresistible to raise the complexity of the system. The beam-slider system is a special structure that consists of a beam and a mass that can slide along it. There is a passive method in the beam-slider structure that can overcome the drawbacks. When the structure is excited, the beam vibration produces an inertial force that drives the slider to move transversely along the beam. The slider movement changes the mass distribution of the entire structure, which, in turn, affects the dynamics of the beam. That means there is a coupling between the beam vibration and the slider motion, which is achieved by the internal forces within the structure, independent of any external factors. This passive technique of modifying the structural parameters is the key to implementing passive control. What is more interesting is that the changes in these parameters are directly related to the beam vibration state, suggesting no necessity to keep track of the beam state. The structural parameters will adjust automatically in response to changes in the beam vibration state.

Miller et al. [18] experimentally investigated the passive self-tuning behavior of a clamped-clamped beam-slider structure. The results revealed that the structure had a promising potential for broadband vibration energy harvesting applications. The clamped-clamped beam-slider structure was studied from different aspects by different researchers [19–24], and the results have proven that its excellent passive self-tuning behavior can enable a vibration energy harvester to achieve a wide bandwidth. Recently, Krack et al. [25] established a theoretical model to explain the passive self-tuning behavior in beam-slider structures. They showed that the clearance and friction of the slider played crucial roles in affecting the complex dynamics of beam-slider structures, including chaotic motions, transient resonance, hysteresis effects, and the coexistence of multiple steady vibration states. The latest work from their group [26] considered the boundary conditions of the clamped-clamped beam as nonideal. The simulation results agreed with the experiments, including the slider moving speed, beam vibration amplitude, and final positions of the slider. Lan et al. [27] investigated the behavior of a frequency-self-tracking energy harvester based on a linear beam-slider structure. During the linear beam vibration, the sliding mass on the cantilever beam could adjust its position automatically. The slider movement changed the second natural frequency of the structure to match the excitation frequency, ensuring the structural resonance to maintain the high efficiency of the energy harvester. Wang et al. [28] attached a pair of magnetic cylinders to the top and bottom surfaces of a cantilever beam and used them as a slider. The collision between the slider and the beam was avoided by eliminating the gap. The experimental results showed that the structure could maintain resonance over the frequency range of 6–9 Hz, and the output power could power commercial LEDs or a digital clock. Soltani and Rezazadeh [29] investigated a hybrid clamped-clamped beam-slider structure with passive and active tuning methods. The slider's position on the beam could be changed passively or controlled actively by a motor with a cable. The work focused on the tuning frequency ranges, and the system could realize self-tuning by two methods. The system did not refer to the bistable system and its chaotic behaviors. The transfers of vibration energy orbits were not discussed in detail as well. Bukhari et al. [30] investigated an electromagnetic vibration energy harvester consisting of a linear beam and a sliding mass. The free movable slider could self-adaptively settle down on a vibration node or antivibration node. As a result, the harvester with a sliding mass had significantly improved bandwidth and efficiency compared to that with a fixed mass. The chaotic vibration phenomena did not appear in the system.

In previous works [31,32], researchers have studied the self-tuning capability of nonlinear beam-slider structures. The hardening nonlinearity could broaden the structural resonance bandwidth, and the slider movement could help the beam capture high-energy orbits. The process detailed earlier is based on mechanical coupling and is devoid of any assistance from external devices. This passive control method has promising advantages in energy harvesting applications. For the beam-slider energy harvester presented in this article, two key modifications are introduced to differentiate its structure from those documented in the existing literature. First, the auxiliary magnets are reconfigured to transform the beam's hardening monostable nonlinearity into bistable nonlinearity. The chaotic vibration then becomes one of the most detrimental factors affecting efficiency. This article endeavors to tackle the issue of transferring vibrations from chaos to inter-well when multiple behaviors coexist. Second, additional magnets are affixed to the slider to exert repulsive forces along the beam, serving as a substitute for the gravitational force induced by the inclination in previous systems. This approach facilitates practical implementation. This article will investigate the dynamics of a bistable beam-slider energy harvester and explore a passive method to achieve chaos-to-inter-well vibration transfer. The remaining of the article is structured as follows. Section 2 offers an introduction to the system's working principle. The experimental details are elaborated in Sec. 3. The findings from the experiments are presented and discussed in Sec. 4. A summary of the conclusions is provided in the final section, Sec. 5.

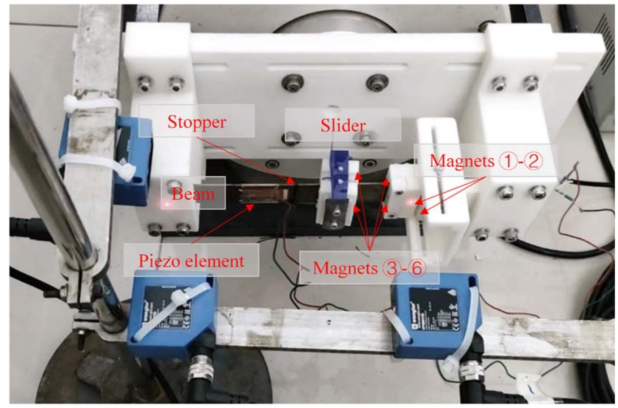

Fig. 2 Physical prototype of the bistable beam-slider energy harvester

2 Working Principle

The bistable beam-slider structure diagram is shown in Fig. 1. The beam-slider structure comprises a cantilever beam, six magnets, and a movable mass (slider). The magnets are numbered ①–⑥. Magnets ① and ② are affixed, respectively, on the free end of the beam and the primary structure with the same magnetic poles facing each other. The repulsive force generated by the two magnets with an appropriate distance introduces the bistable nonlinearity to the beam dynamics. When the primary structure is mounted on the shaker that produces an excitation, the beam may vibrate within a single well or across wells, which are known, respectively, as inner- and inter-well vibrations. The chaotic vibration refers to a complex, unpredictable, and irregular behavior that does not follow a repeatable pattern. The inter-well vibration is favorable for energy harvesting due to its steady and large amplitude. The slider can move along the beam. Eight bearings are embedded in the slider to ensure line contact with the beam to reduce friction. When the beam is vibrating, on the one hand, the slider can vibrate together with the beam in the transverse direction. On the other hand, the slider can move along the beam due to the inertial force. Besides, the magnets ③–⑥ on the free end and the slider provide symmetrical repulsive forces. The inertial and magnetic forces on the slider are determined by the vibration amplitude and slider position, respectively. And they are of comparable magnitude but in opposite directions. The stopper on the beam restricts the movement of the slider, thereby protecting the piezoelectric material at the clamped end that converts kinetic energy to electricity.

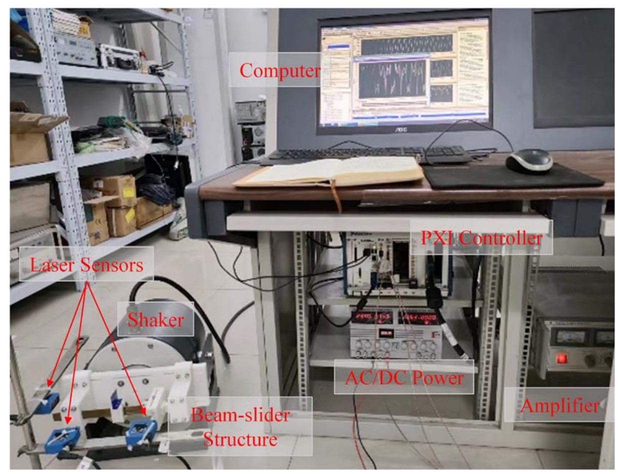

Fig. 3 Experimental setup

3 Experimental Setup and Methodology

3.1 Experimental Setup. The prototyped beam-slider energy harvester and the experimental setup are shown in Figs. 2 and 3, respectively. The beam is made of bronze and fixed on the 3D-printed primary structure. The repulsive force that depends on the distance of two magnets can introduce an axial force to the beam and, thus, a bistable nonlinearity to the system. The whole structure is mounted on the shaker, which generates vibrations in the horizontal direction to eliminate the influence of gravity. Three auxiliary laser sensors are used to support the conduction of the experiment. One is used to measure the base displacement to provide the feedback signal for the closed-loop control. The other two laser sensors are to measure the displacements of the beam and the slider, respectively. The measurement data are acquired and analyzed by a data acquisition system (NI PXI

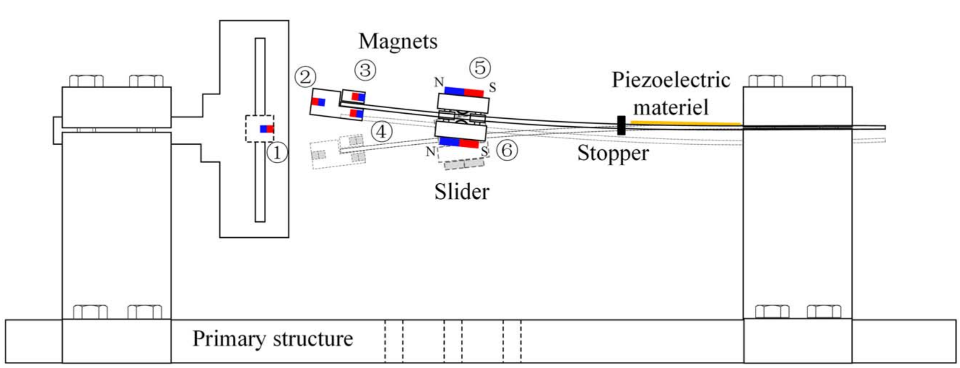

Fig. 1 The diagram of the bistable beam-slider energy harvester

Table 1 Geometric and material parameters of the manufactured physical prototype

Parameters	Symbol	Value
Slider mass	m_s	0.042 (kg)
Tip mass of the cantilever beam	m_t	0.019 (kg)
Beam Young's module	E	113 (GPa)
Beam mass density	ρ	8900 (kg/m^3)
Beam dimensions	$L \times B \times h$	$135 \times 20 \times 1.5$ (mm)
Magnets dimensions	$L_m \times B_m \times h_m$	$20 \times 5 \times 1.5$ (mm)
Magnet residual flux density	B_r	1.3 (T)
Distance of magnets ① and ②	D_1	6 (mm)
Distance of magnets ③ and ④	D_2	12 (mm)
Distance of magnets ⑤ and ⑥	D_3	12 (mm)
Base excitation level	A_0	3.92 (m/s^2)
Piezoelectric element dimension	$L_p \times B_p \times h_p$	$28 \times 20 \times 0.27$ (mm)
Permittivity component at constant strain	ε_{33}^s	12.653 (nF/m)
Strain coefficient of the piezoelectric layer	d_{31}	−170 (pC/N)

4072). Specific parameters of the manufactured physical prototype are listed in Table 1.

3.2 Experimental Methodology. We designed two experiments to test and evaluate the dynamics of the beam-slider harvester. In the first experiment, the slider is fixed at the free end of the beam or near the stopper. The excitation frequency sweeps forward and backward. The responses of the beam are recorded. The slider is set free in the second test, and a sinusoidal dwell excitation is applied. The displacement response of the beam and the trajectory of the slider are measured. Once the beam vibration

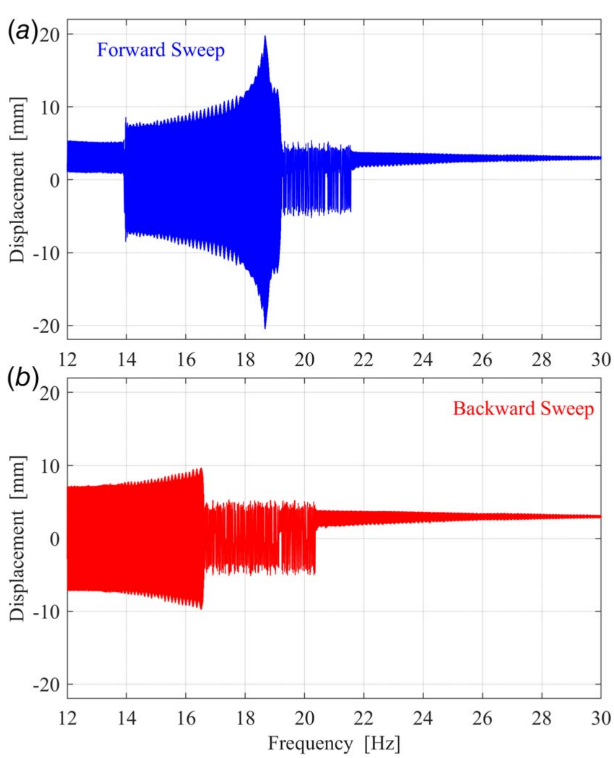

Fig. 4 (a) Forward and (b) backward sweep frequency responses of the bistable beam-slider harvester with the slider fixed at the free end

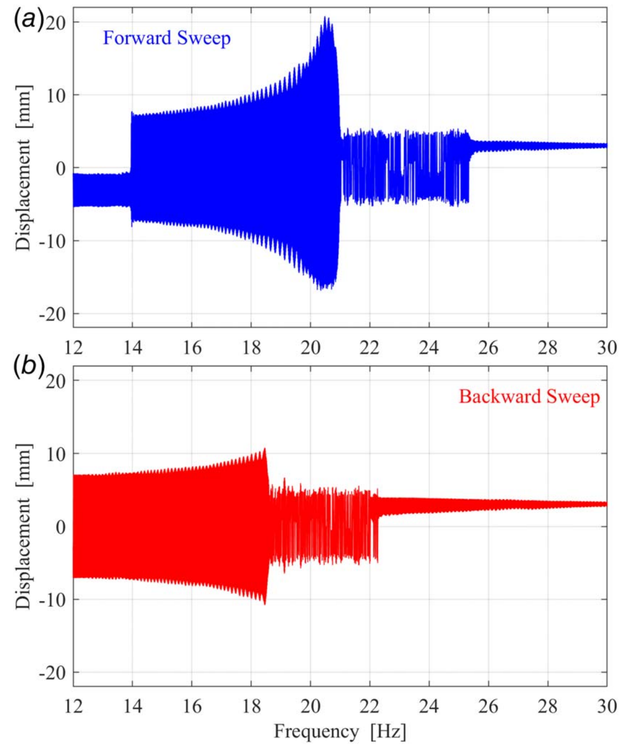

Fig. 5 (a) Forward and (b) backward sweep frequency responses of the bistable beam-slider harvester with the slider fixed near the stopper

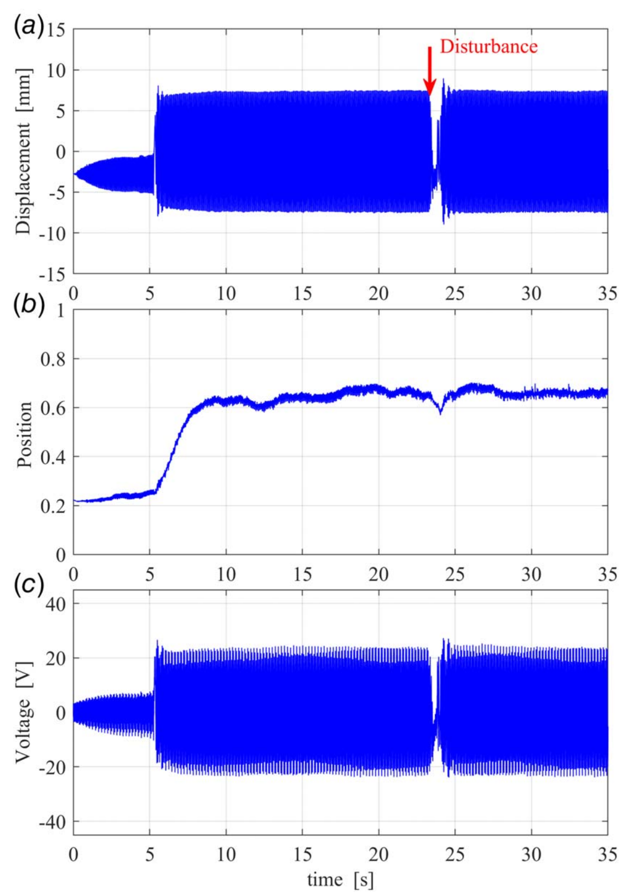

Fig. 6 Time responses of the beam-slider energy harvester with a freely movable slider under the base excitation frequency of 15 Hz: (a) beam response, (b) slider position, and (c) output voltage

stabilizes on the high-energy orbit, a disturbance is introduced to verify whether this preferred state can be sustained. In the experiment, the pulse and disturbance are two different types of actions applied to the system. Pulses are short shocks to deviate the bistable beam away from the equilibrium position. By applying a pulse, kinetic energy is injected into the bistable system to help the harvester cross the barrier between two wells. Unlike pulses, disturbances are obstructions applied to the harvester. When the bistable beam vibrates in the inter-well energy orbit with a large amplitude, disturbances act as instant obstacle forces, attempting to prevent the beam from remaining near the equilibrium point. Both the pulses and disturbances in the experiment are produced by manual short-time mechanical shocks.

4 Experimental Results

4.1 Response of the Bistable Configuration. Given the slider is fixed at the free end, the forward and backward frequency responses of the bistable beam are shown in Figs. 4(*a*) and 4(*b*), respectively. Both responses exhibited three types of vibration phenomena. The first type is the small amplitude inner-well vibration with an equalization point of ±2.5 mm. The second type is the inter-well vibration, which is symmetric to zero (*x*-axis) and has an amplitude greater than 5 mm. The third is the chaotic response, involving unpredictable and random jumps between the two potential wells. In the two responses presented in Figs. 4(*a*) and 4(*b*), the three vibration types appear in different frequency ranges, and within certain ranges, they even coexist. The orbit in which the bistable beam vibrates depends on the initial conditions. Upon comparison, it can be observed that both inner-well and inter-well vibrations coexist below 14 Hz. Chaotic and inter-well vibrations coexist in the frequency range of 16.2–19 Hz. When the frequency is above 19 Hz, the inter-wall vibrations disappear, and only chaotic vibrations remain. Over the frequency range of 19–22 Hz, inner-well and chaotic vibrations coexist. It is well known that the inter-well vibration orbit is highly desired to ensure the high efficiency of the energy harvester. Realizing the orbit transfer from chaos to inter-well vibration in the frequency range of 16.2–19 Hz is a formidable but meaningful task.

When the fixed position of the slider changes to the stopper, the equivalent mass of the structure decreases. Thus, the whole response shifts to the higher frequency range, as shown in Figs. 5(*a*) and 5(*b*). The frequency band of inter-well vibration in the forward sweep expands from 14–19 Hz to 14–21 Hz. And the upper bound of the chaotic range extends to about 25 Hz. In the backward sweep response, the chaotic vibration transition takes place around 18 Hz and 22 Hz. It is noted that the vibration types of the bistable beam do not change, even though the slider position is changed. Only the different types of vibration appear in different frequency ranges. The results in Figs. 4 and 5 represent two extreme cases with the slider position at the free end and near the stopper. The frequency responses alteration will be intermediate if the slider is placed somewhere in between.

4.2 Response of the Self-Adaptive Configuration. This subsection presents the evaluation results of the self-adaptive configuration that refers to the beam-slider energy harvester with a freely movable slider. In the evaluation test, a dwell harmonic

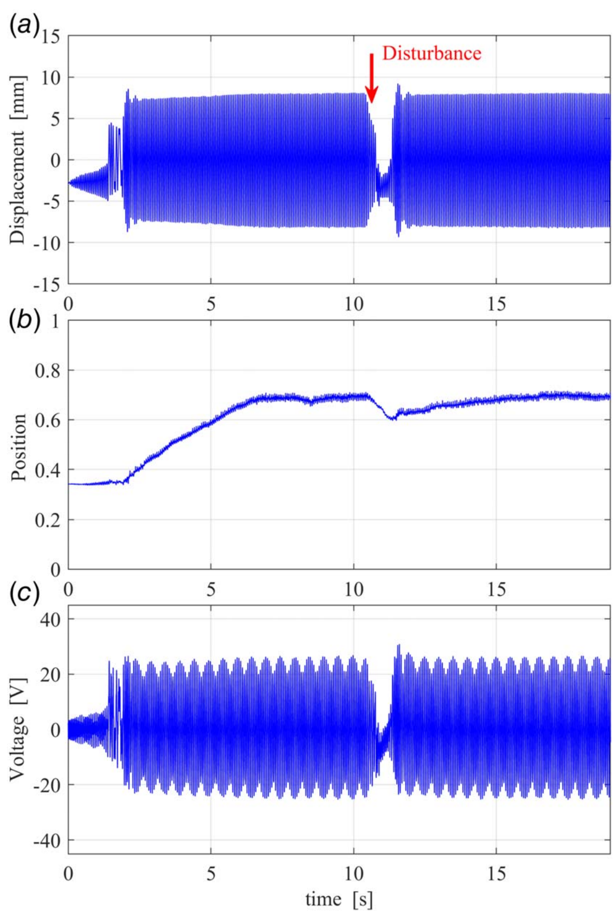

Fig. 7 Time responses of the beam-slider energy harvester with a freely movable slider under the base excitation frequency of 16 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

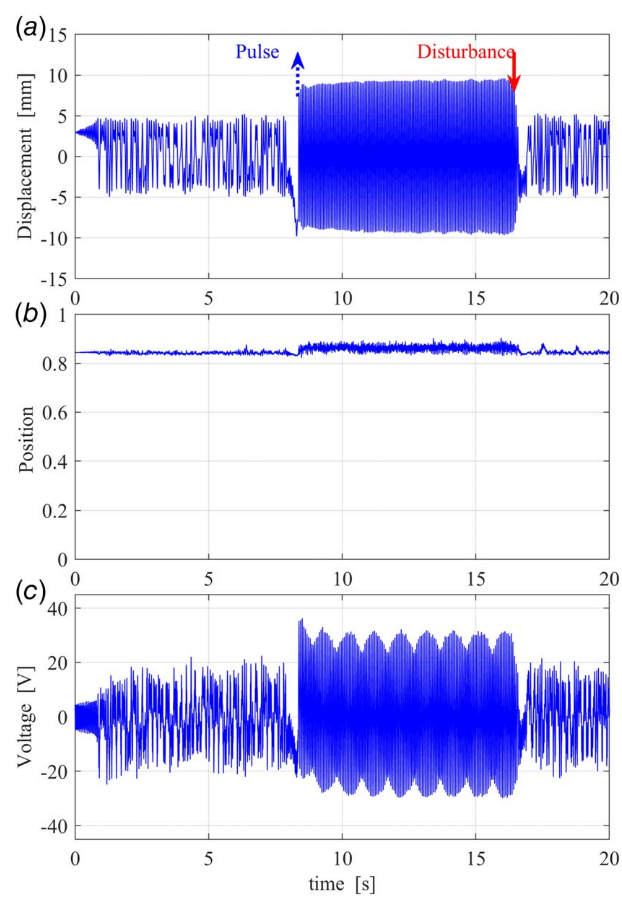

Fig. 8 Time responses of the bistable energy harvester with a fixed slider near the stopper under the base excitation frequency of 17 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

excitation is applied to the self-adaptive harvester. The system is initially in the static state, where the beam is at equilibrium in one well, and the slider rests at a casual position between the free end and the stopper. Given the frequency dwells at the frequency of 15 Hz, the beam displacement, the slider position (0 and 1 indicate the positions at the stopper and the free end, respectively), and the open circuit voltage are presented in Figs. 6(*a*)–6(*c*), respectively.

When the slider is initially set at 0.2, the nonlinear beam vibrates in the inner-well orbit around the position of −2.5 mm at the beginning. The amplitude is relatively small, just about 4.2 mm. At this position, the magnetic force on the slider is relatively small due to the large distance between the magnets. The inertial force on the slider caused by the beam vibration drives the slider to move toward the free end. This movement changes the mass distribution of the structure and triggers the jump to the inter-well orbit, increasing the vibration amplitude significantly. The inertial force on the slider increases and drives the slider to move faster toward the free end. The magnetic force also increases because of the decreased distance between the magnets on the slider and the free end. Finally, the two forces are balanced at about 0.65. The slider stops around the balanced position with a slight shaking motion. The bistable beam attains the steady-state inter-well vibration. Though the system is perturbated by an external disturbance at 24 s, the inter-well vibration can recover autonomously. Only a minor alteration is observed in the slider position of Fig. 6(*b*) due to the fast rebuild of the inter-well vibration. As shown in Fig. 6(*c*), the output voltage of the piezoelectric element reaches 24 V when the structure is in the inter-well vibration state, which is about three times larger than that in the inner-well vibration state.

As shown in Fig. 7, when the excitation frequency rises to 16 Hz, the nonlinear beam can quickly enter the inter-well energy orbit at the beginning. The quasi-steady-state beam vibration amplitude is about 8.9 mm. The inner-well vibration does not appear as shown in Fig. 6(*a*). The larger amplitude vibration supplies the slider with a larger inertial force, driving it to move faster toward a slightly farther balance point of 0.7. After the disturbance, the slider first moves about 5.6 mm toward the clamped side before reversing. Similar to the results at 15 Hz, the energy harvester under the excitation frequency of 16 Hz can resist external disturbance and maintain a high output voltage of about 27 V.

When the excitation frequency dwells at 17 Hz, the results differ from the previous two cases. Given the slider is fixed near the free end (degrades to the conventional bistable configuration without self-adaptivity), approximately at 0.85, the nonlinear beam vibrates chaotically, as shown in Fig. 8(*a*). The slider remains at a constant position, as depicted in Fig. 8(*b*). Until the external pulse is injected (marked by the dashed arrow in Fig. 8(*a*)), which inputs energy into the system, the nonlinear beam can transfer the state from chaotic to inter-well vibration. However, an external disturbance (marked by the solid arrow in Fig. 8(*a*)) can easily disrupt this desired vibration state. The efficiency of the harvester in chaos decreases. Thus, the energy harvesting performance is unreliable when chaos coexists with inter-well vibration at this excitation frequency. The result in Fig. 8 reveals the typical problem for most conventional bistable energy harvesters: obtaining and maintaining the inter-well vibration is difficult and challenging.

As shown in Fig. 9, when the slider is set free at the same frequency of 17 Hz, the nonlinear beam vibrates in chaos as the previous at the beginning. As the slider moves toward the free end,

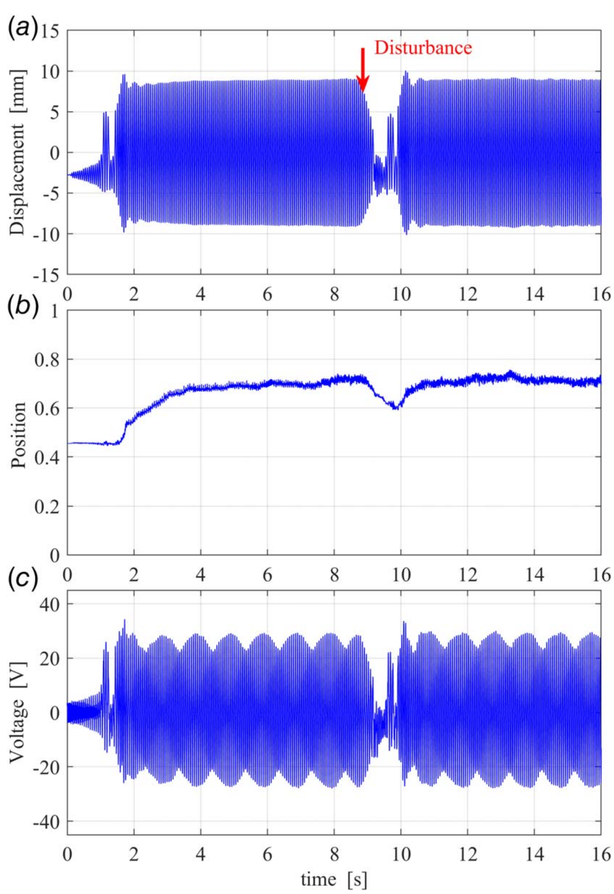

Fig. 9 Time responses of the beam-slider energy harvester with a freely movable slider under the base excitation frequency of 17 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

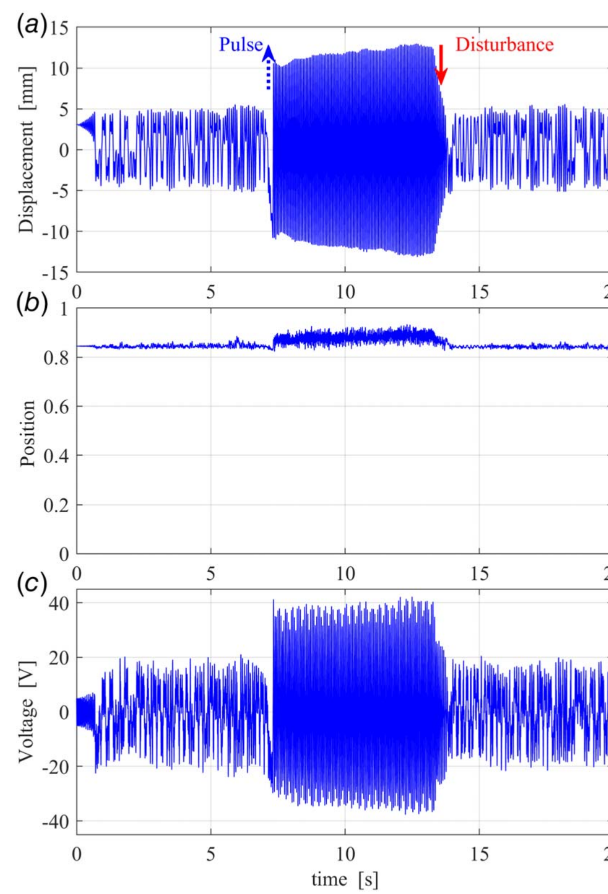

Fig. 10 Time responses of the bistable energy harvester with a slider fixed at about 0.82 under the base excitation frequency of 18 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

driven by the inertial force, the chaotic vibration automatically transfers to the inter-well vibration. The increase in the beam response pushes the slider closer to the free end. Finally, the magnetic forces and the inertial force get balanced around the position of 0.72. When the external disturbance is introduced, the inter-well vibration changes into chaos for a short time. The decrease in the vibration amplitude then causes a decrease in the inertial force. The magnetic force takes over the domination and drives the slider back to the clamped side. This movement also weakens the magnetic force on the slider due to the increase in the distance between the magnets. At the point about 0.6, the inertial force surpasses the magnetic force. The slider reverses its moving direction and helps the system recover the high-energy orbit. As a result, the inter-well vibration of the beam can always be sustained. The output voltage can be kept around 29 V. Since the slider and the bistable beam are mechanically coupled, the control process is passive and adaptive. The slider position on the beam changes according to the beam vibration state. By this mechanism, the energy harvester can automatically realize the transformation from chaotic to inter-well vibration without energy consumption and the need for external assistance.

A similar situation also happens when the frequency dwells at 18 Hz. As the slider position is fixed at about 0.82 around the free end in Fig. 10, the nonlinear beam vibration contains two states: chaotic and inter-well vibration. An external pulse is needed to help the system jump to the inter-well energy orbit. Hence, this harvester is not reliable when confronted with an external disturbance. When the slider is set free in Fig. 11, the energy harvester can realize a passive and adaptive transformation from chaos to inter-well vibration. The difference between Figs. 9(*a*) and 11(*a*) is that the bistable beam has a larger vibration amplitude after the slider stopping. That is also why the slider balance position (0.82) gets closer to the free end, and the output voltage of the energy harvester peaks at 40 V. The repulsive magnets with closer distance also offer more potential energy to the slider. The reaction of the slider in Fig. 11(*b*) after the disturbance is more distinct.

When the excitation frequency increases to 19 Hz, the chaotic and inter-well vibrations coexist, as demonstrated in Fig. 12. It is worth noting that 19 Hz is very close to the jump-down frequency, as shown in Fig. 4(*a*). Even though the slider approaches the free end under the inertial force, the beam vibration fails to keep at the inter-well vibration. The temporary increase of the beam vibration injects momentum into the slider. After the kinetic energy of the slider runs out, the magnetic force drives the slider to move to the clamped side gradually until it stops at the balance point of 0.5. When the excitation frequency increases to 20 Hz, only chaotic vibration exists, as can be observed in Fig. 13. Though the slider automatically moves from 0.2 to 0.5, the chaotic vibration state of the energy harvester does not change at all. The aforementioned results indicate that the two frequencies, 19 Hz and 20 Hz, are out of the self-adaptive range of the beam-slider energy harvester.

Root-mean-square (RMS) voltage is a figure of merit often used to evaluate the efficiency of an energy harvester. The RMS voltage outputs of the beam-slider and fixed-mass energy harvesters are compared in Fig. 14. The voltage output of the fixed-mass bistable energy harvester bifurcates into two branches due to the existence of two states, i.e., inter-well and chaotic. The voltage produced during inter-well vibration significantly surpasses the voltage generated

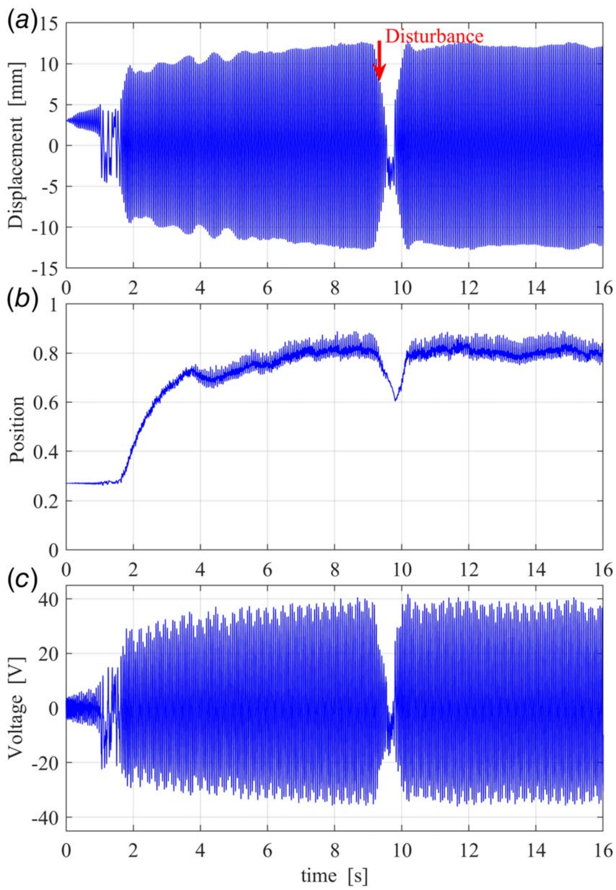

Fig. 11 Time responses of the beam-slider energy harvester with a freely movable slider under the base excitation frequency of 18 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

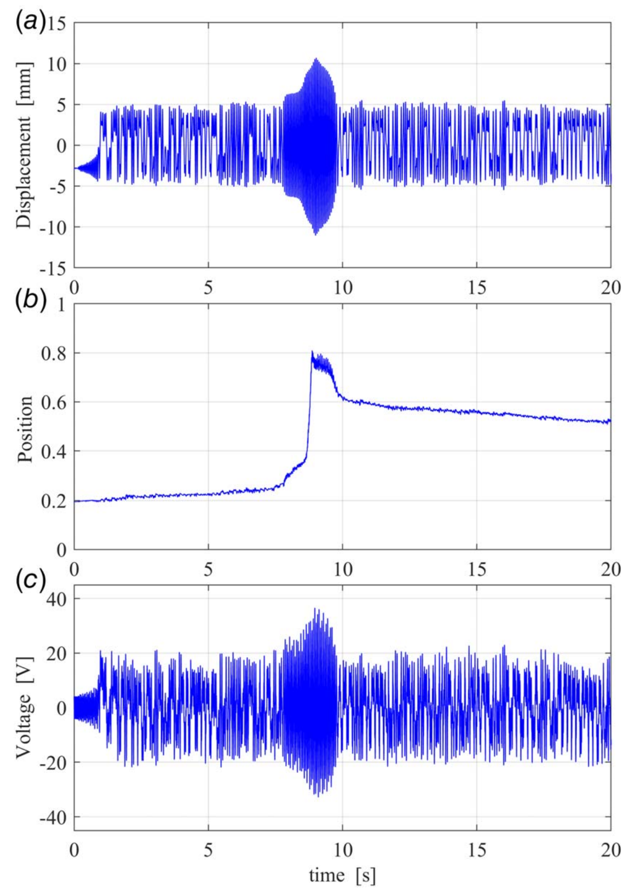

Fig. 12 Time responses of the beam-slider energy harvester with a freely movable slider under the base excitation frequency of 19 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

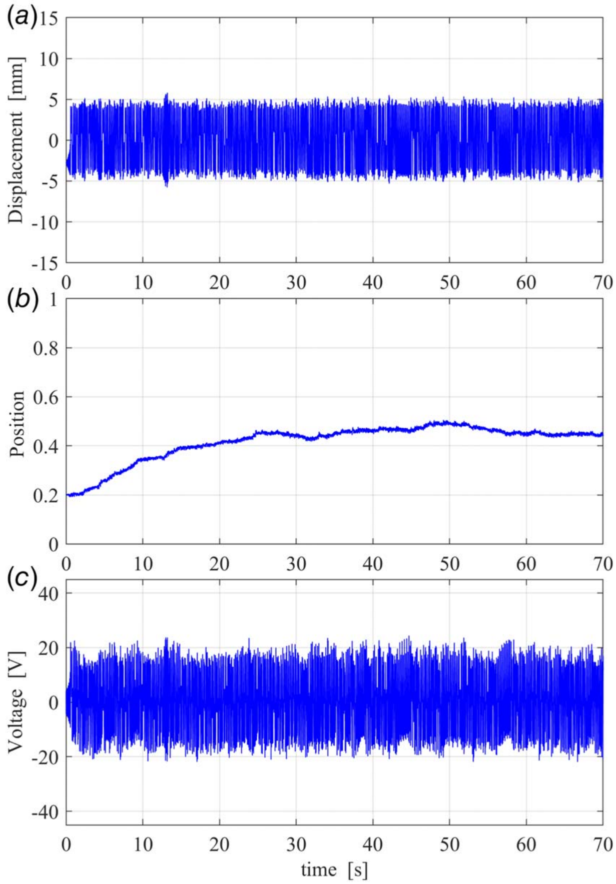

Fig. 13 Time responses of the beam-slider energy harvester with a freely movable slider under the base excitation frequency of 20 Hz: (*a*) beam response, (*b*) slider position, and (*c*) output voltage

during chaotic vibration over the frequency range of 16–18 Hz. Since the voltage may decrease significantly when transitioning to the chaotic branch due to disturbances, as shown in Figs. 8 and 10, the fixe-mass harvester is unreliable. As for the beam-slider

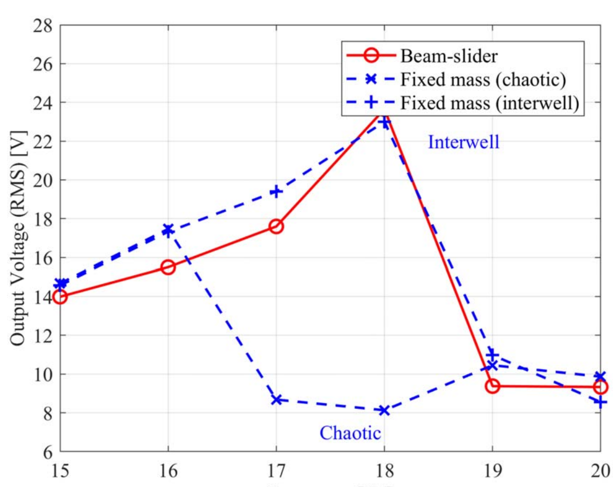

Fig. 14 The comparison of the RMS voltage outputs of the bistable beam-slider energy harvester and the fixed mass (in chaotic/interwell orbits) energy harvester. Note that the mass is fixed at the proper position to trigger the resonance of the beam. If the mass is placed at a different location, the fixed-mass harvester will operate in an off-resonance state across the frequency range under investigation.

energy harvester, the chaos in the coexisting frequency range is destabilized by the passive chaos-to-inter-well vibration transfer, as shown in Figs. 9 and 11. It also eliminates the sensitivity to initial conditions because of its adaptive adjustment capacity. In general, the slider on the nonlinear beam helps prevent the bistable beam from running in a chaotic state. The notable advantage of the beam-slider harvester over the fixed-mass counterpart is that the bistable system can consistently operate in the desired inter-well energy orbit without being affected by disturbances and initial conditions. Moreover, it should be noted that the fixed-mass harvester used for comparison is an optimal one, with the mass carefully placed at the proper location to trigger the structural resonance. If the mass is arbitrarily placed on the beam, the fixed-mass harvester will operate entirely in an off-resonance state, resulting in minimal voltage output.

5 Conclusion

This article has experimentally studied the dynamics of a bistable vibration energy harvester based on a beam-slider structure. When the beam vibrates, it prompts the slider to vibrate in a transverse direction. The inertial force due to the beam vibration and the magnetic force on the slider are of comparable magnitudes but in opposite directions. The beam vibration amplitude and the slider position, respectively, determine the inertial and magnetic forces. If the system operates in a high-energy orbit (inter-well vibration), the slider will stop at the position where two forces are balanced. As the system jumps to the low-energy orbit because of any external disturbance, the magnetic force will surpass the inertial force and drive the slider toward the clamped side. The slider movement changes the mass distribution of the structure and helps the nonlinear beam recover the high-energy orbit motion. The experimental results show that the energy harvester successfully transformed the chaos into inter-well vibration over the frequency range of 16–18 Hz. The beam-slider structure proposed in this article has demonstrated significant improvement when it operates in the inter-well vibration mode. However, the stability issue cannot be neglected due to the disturbances in practical application scenarios. It is worth noting that achieving the transition from chaos to inter-well vibration is essential for the bistable beam-slider harvester to uphold its high efficiency. Compared to the active methods with numerous drawbacks in practice, the beam-slider structure offers a passive solution to address this issue. The whole process operates passively and automatically without the need for any additional devices or energy consumption. Overall, the experimental data show that the self-adaptive system outperforms the fixed-mass bistable energy harvester in terms of reliability and resistance to disturbances.

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

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